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**A POSITIVE PROGRAMMING APPROACH TO VALIDATION AND
CALIBRATION IN AGRICULTURAL SECTOR MODELS:
THE CASES OF THE TURKISH NATIONAL AND
CALIFORNIA REGIONAL MODELS**

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Haluk Kasnakoglu
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University of California, Davis
Department of Agricultural Economics

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HALUK KASNAKOGLU*

and

RICHARD E. HOWITT*

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*Associate Professors of Economics and Agricultural Economics at the Middle East Technical University in Ankara, Turkey and University of California at Davis, California, respectively. Authorship seniority is not assigned.

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I. INTRODUCTION

The rapid development of computers and efficient solution algorithms have made the extensive use of large-scale price-endogenous programming models possible by economists to simulate the impact of farm programs upon the agricultural sector. Policy makers and many economists on the other hand, have been reluctant to rely heavily on programming models for planning, due to the poor performance of these models at disaggregated levels and the lack of widely accepted validation procedures.

Recently efforts to make programming models produce results closer to those actually observed have developed in two directions: The first set of approaches stemmed from the recognition that the dimension of the optimal solution to a linear programming problem is equal to the number of binding constraints at the optimum. These approaches involve modifications in the constraint set.¹ Specifically they include the introduction of flexibility or capacity constraint (Jabara and Thompson [1980]; Norton and Solis [1983]; Sharples and Schaller [1968]; McCarl [1982]; Le-Si, Scandizzo and Kasnakoglu [1983]), and rotation activities or production plans instead of single crop activities (Duloy and Norton [1983]; Le-Si, Scandizzo and Kasnakoglu [1983]; Kutcher and Scandizzo [1981]; Egbert and Kim [1975]; Meister, Chen and Heady [1978]). These approaches often resulted in models that are tightly constrained, which could only produce that subset of normative results that the calibration constraints dictated, and hence are inappropriate under policy changes or for projections into the future.² The second set of approaches modify the objective function. It was recognized that the linearities of the

objective function in output or other decision variables had to be avoided to solve the problem of over-specialization. Consequently, nonlinearities are introduced into the revenue part of the objective functions with downward sloping demand functions (Duloy and Norton [1981]) and to the cost part of the objective function with risk (Freund [1956]; Hazell and Scandizzo [1974]). The use of these techniques has also been facilitated with several ingenious approaches to approximate the resulting nonlinear objective functions while maintaining the convenient algorithmic properties of linear programming (Norton and Solis [1983]; Kutcher and Scandizzo [1981]; Le-Si, Scandizzo and Kasnakoglu [1983]). A serious drawback to the implementation of the above stagewise techniques has been the lack of detailed data on the technology at the microeconomic level. Furthermore, their calibration contributions are more appealing than their theoretical properties. Thus, little attention was given to studies which attempted to improve the theoretical basis of these concepts (Paris [1979]; Wicks [1978]). Rather, the sector modelling literature has employed the demand and risk parameters (i.e., elasticities and risk aversion coefficients) as calibration tools (Pomareda and Simmons [1983]; Kutcher and Scandizzo [1981]; Adams, Johnston and King [1978]; Le-Si, Scandizzo and Kasnakoglu [1983]).

Agricultural sector models following one or more of the methods discussed above are usually subjected to "calibration," "verification" and "validation" tests.³ These tests fall under four broad categories:

1. First: Comparing the principal variables of the base solution with observed data in the base year. Most of the tests performed on sector models in the literature fall under this category. They take the form of "capacity tests" (Kutcher [1983]; Bassoco and Norton [1983]) or "consistency tests" (Kasnakoglu and Howitt [1985]) which check model feasibility and consistency

by forcing the model to reproduce the base year magnitudes, and "goodness of fit tests" which compare the simulated base solution variables such as area, production, prices, trade, etc., with observed evidence using Theil's U coefficients or regressions (Duloy and Norton [1983]; Kutcher [1983]; Bassoco and Norton [1983]; Pomareda and Simmons [1983]; Kutcher and Scandizzo [1981]; Egbert and Kim [1975]; Adams, Johnston and King [1978]; Le-Si, Scandizzo and Kasnakoglu [1983]; Jabara and Thompson [1980]).

ii. Second: Confronting the estimates implied by the base solution with theory, actual evidence or with the results of econometric studies. Examples in this category are implied supply function tests (Kutcher [1983]; Shumway and Chang [1977]), and shadow prices for land inputs, (Bassoco and Norton [1983]).

iii. Third: Testing the validity of the assumptions of the model. These tests are usually applied to the perfectly competitive market and price-endogeneity assumptions central to most programming models (Kutcher [1983]; Bassoco and Norton [1983]).

iv. Fourth: Ex-post projections of the base solution forwards or backwards to a year other than the base period and comparing the simulated variables with observed variables in the projected year (Nugent [1970]; Kasnakoglu and Howitt [1985]).

From the view point of a policy maker a model's value must be dominated by its ability to predict the reactions of the economic sector to changes in exogenous or policy parameters. Thus, the test of a model's value and validity for policy purposes should be based on its ability to predict the reaction to changes that occur outside the base period. Under this criterion, only the fourth method can be defined as validation and the first three methods are specified as calibration or estimation. Calibration of the model

to the base year parameters is a necessary but not sufficient condition for validation by prediction. Perhaps one of the reasons why so few agricultural sector model researchers have been concerned with validation by prediction, is that the necessary condition of calibration against base year parameters has posed substantial difficulties.

In the section that follows, results from using the PQP method of model construction and calibration are discussed. In addition, tests of validation by prediction are undertaken for two types of sector models.

II. A POSITIVE QUADRATIC PROGRAMMING APPROACH TO CALIBRATION AND VALIDATION

The method termed Positive Quadratic Programming (PQP), (Howitt and Mean [1985]) amends normative linear and nonlinear microeconomic models by a positive measure of the nonlinear part of the cost function. This cost is calculated from the discrepancy between the crop average value product implicit in the linear cost specification and the first order conditions implied by the observed crop allocation decisions.

In short, the farmer's aggregate crop allocation decisions in a region are used to calculate additional nonlinear cost terms that would result in the observed allocations, rather than adding constraints on the linear system that would force the allocations.

Using this positive approach, the linear model can be exactly calibrated to observed outputs for a single year or calibrated with a least-squares criterion if actual crop acreages for several years are known. The resulting optimization problem incorporates a quadratic cost term for each regional crop grown and is constrained only by those constraints that can be empirically justified. The problem is solved as a quadratic programming problem.

The additional PQP cost component is termed the implicit cost since it is implied in a positive sense by the farmer's crop allocations.

Empirical implementation of positive programming is achieved in two stages. The first stage starts with the data and specification of a conventional LP (or QP) problem. The actual regional crop acreages (\tilde{x}) are increased by a small perturbation ϵ consistent with (Howitt and Mean [1985]) Theorem I, say $(.001) \tilde{x}$, and are formulated as upper bound inequality constraints. The constrained LP problem is now run to obtain the dual values on the calibration constraints for the $n-m$ crops at interior optima. The ϵ perturbation of the calibration constraint right hand side ensures that relevant resource constraints will be binding on the resource constrained crops in the basis.

Although it would be preferable to estimate the quadratic production function coefficients for the constrained crops, they are neither required nor possible for the single time period case.

The vector of $(k-m)$ dual values from the first stage problem for the interior crops is multiplied by the negative reciprocal of the observed acreages \tilde{x}_i $i=1 \dots k-m$ and used as the diagonal coefficients of the quadratic cost function in the second stage problem. The second stage problem is then solved for the optimal base period solution. The principal steps are:

- a Given a standard LP or QP and the vector of actual acreage grown \tilde{x} . Perturb \tilde{x} by ϵ and add the calibration constraints.
- b Run the first stage problem. The observed crop vector, \tilde{x} is $k \times 1$ ($k > m$), therefore the first stage will result in m binding resource constraints, and $k-m$ dual values corresponding to the binding calibration constraints.

c If the production function is quadratic in land and separable, the implicit cost function is quadratic in x , and has the form $1/2\tilde{x}^T E \tilde{x}$ where E is a $(k-m) \times (k-m)$ positive semidefinite matrix. By the PQP theorem II (Howitt and Mein)

$$-\lambda^* = E\tilde{x}$$

Given the minimal data set \tilde{x} , cross cost effects are restricted to zero, and thus for the single period calibration case considered here E is a diagonal matrix with nonzero elements e_{ii} where:

$$e_{ii} = -\lambda_i^* / \tilde{x}_i$$

corresponding to the interior cropping activities.

d Using the values e_{ii} , the second stage problem is specified as

$$\text{Max } f(x) + 1/2x'Ex$$

$$\text{Subject to } Ax \leq b \quad x \geq 0$$

The second stage problem calibrates exactly with the base year vector \tilde{x} without additional constraints, and is available for policy analysis in the knowledge that the model response will be determined by economic comparative advantage and resource constraints that have a clearly demonstrated empirical basis.

While the ability to develop exactly calibrated models for a single year without adding constraints is an advance, the policy value of such models depends on the ability of the updated model to represent future years. In the remainder of the paper, the PQP approach is applied to Turkish Agricultural Sector Model (TASM) which is an aggregate national model and California Agricultural Resources Model (CARM) which is a regional model. In the case of TASM, the model, augmented with PQP terms, is employed to project changes in area, production and consumption patterns two years ahead of the base year. In the case of CARM, the PQP terms from eight years of base solutions, are

used in an econometric specification to estimate the dynamic and stochastic nature of regional crop acreage response.

III. THE TURKISH AGRICULTURAL SECTOR MODEL (TASM)

a. The Basic Structure of TASM

TASM is partial equilibrium, static, optimization model to simulate the agricultural sector and resource allocation effects of agricultural policies on production, consumption and trade patterns.

The objective function maximized in the model is the sum of consumers' and producers' surplus, plus net export revenue, and minus the labor reservation wage. Risk costs are included as part of production within E-V framework.⁴ Given the structure of price responsive consumer demands, production activities and trade possibilities, optimality entails equating supply to domestic plus foreign demand, and prices to marginal costs for all commodities, making provisions for risk and allowing for the reservation wages.

The core of the model consists of the production activities and resource constraints. The input and output coefficients for single, multiple, and rotation crop production activities are specified for each unit of land. In addition to land, other input requirements for production are labor, tractor, fertilizers, animal power, seed and capital. Animal power is supplied by livestock production activities, and seed is supplied by crop production activities. The model is given a choice of two production techniques, namely mechanized and non-mechanized. It can assign any combination of weights to these two techniques to produce a single crop, as required by the optimal allocation of resources.

The livestock subsector works similarly to the crop sector. The explicit production cost for animal husbandry is labor. Other inputs required are cereals, straws and forage which are by-products of crops; and concentrates which are derived from crops processed for human consumption. Pasture land is also required for animal grazing, with the exception of poultry, to supplement livestock feeding. In addition to meat, milk, wool, hide and eggs, the livestock production activities also provide animal power used in crop production activities.

The commodities produced by the production activities are distributed between, i) domestic demand generated through consumer demand functions, ii) demand for cereal used for feeding in livestock sector, iii) demand for seeds used in crop production activities, iv) exports in unprocessed form, v) exports in processed form. On the supply side imports complement the domestic production.⁵

Since generally data available at the farmgate level are the most reliable, prices and some quantities used in the model are incorporated at this level. Import prices and export prices are thus adjusted for transportation and marketing margins. The domestic demand functions are also calculated at the farmgate level.⁶

TASM incorporates 20 annual crops, 15 perennial crops and 20 livestock products, through 33 single annual crops and 15 perennial crop activities, 12 rotations and 25 multiple cropping activities for each production technology and seven livestock activities. Six groups of inputs are incorporated in TASM. Labor, animal power, and tractors are introduced on quarterly basis. Land is classified into treeland, pastureland, and cropland. The cropland is further divided into eight classes distinguishing between various combination of irrigation, temperature and rainfall. Two kinds of

fertilizers, namely, Nitrogen and Phosphate are employed. Input requirements for annual crops are amounts of seed and seedlings, and for perennial crops fixed investment costs are used.

b. Calibration and Validation Tests

Calibration of the 1979 base solution, is performed in two stages. In the first stage, the model is run as a conventional quadratic programming problem, augmented with three sets of PQP constraints: The area constraints, production technology constraint and fallow constraint.⁷ The first stage solution, updated with the results of capacity and consistency tests, was then used as the basis for the second stage solution. The duals on the area, production technology and fallow constraints were transformed as described in section III to PQP terms which were included in the objective function of the second stage as quadratic costs.⁸ The second stage problem augmented with PQP terms and excluding the PQP constraints was run for the 1979 base solution.

The 1979 base year solution, was then employed to project 1981. For this projection, 1979 base year data including yields, demand functions, risk costs, factor costs, exchange rate, trade quantities and prices were updated with ex-post 1981 data or exogeneous projections. It should be noted that a sectoral model should not attempt to predict costs or international trade and prices, but rather predict the reaction of the sector to these changes. The base solution PQP terms were also inflated with changes in GNP deflator and production cost index, for nominal projections with the model. The comparison of the simulated changes in area production and consumption with actual changes between 1979 and 1981 are illustrated in Tables 1-3. With the exception of a few products, TASM has been able to predict changes in direction and magnitudes with no significant bias, and demonstrated itself as

TABLE 1

PERFORMANCE OF TASM IN PREDICTING DIRECTIONS OF CHANGES

Direction Predicted	Area	Percent	Production	Percent	Consumption	Percent
Correct	31	.89	50	.91	53	.96
Incorrect	4	.11	5	.09	2	.04

TABLE 2

PERFORMANCE OF TASM IN PREDICTING ABSOLUTE CHANGES

Percent Error	Area		Production		Consumption	
	Number	Percent	Number	Percent	Number	Percent
< 2	12	.343	25	.456	24	.436
2-4.9	15	.429	17	.309	18	.327
5-10	5	.143	7	.127	7	.127
> 10	3	.086	6	.109	6	.109
Total	35		55		55	

TABLE 3

REGRESSIONS OF ACTUAL CHANGE RATIOS ON PROJECTED RATIOS

	Intercept	Slope	R	N
AREA	.235 (4.69)	.767 (15.87)	.89	33
		.991 (118.3)	.81	33
PRODUCTION	.136 (.90)	.904 (6.79)	.48	51
		1.021 (35.21)	.47	51
CONSUMPTION	.056 (1.24)	.982 (40.49)	.97	53
		1.002 (54.24)	.97	53

Note: two extreme observations in the cases of area and consumption and four extreme observations in the case of production are excluded from the regressions. See Kasnakoglu and Howitt [1985] for a discussion on those products.

a relatively more reliable tool for policy analysis, than its earlier versions without PQP amendment.⁹

IV. THE CALIFORNIA AGRICULTURAL RESOURCES MODEL (CARM)

The CARM model is designed to reflect the effect of changes in input and output prices and changes in the quantity of some resources on agricultural production in California. California agriculture is a complex system of irrigated agriculture producing over 45 field, fodder, vegetable, and fruit crops. Over the 800-mile long irrigated production area there are considerable climatic, fertility, and water availability differences. The heterogeneity of the production regions causes the model to be divided into 14 production regions and covering 44 of the most important crops by acreage and value. This crop and regional disaggregation results in model containing about 600 cropping activities.

Since California has a dominant role in the production of many of the fruit and vegetable crops, the market price is effected by California production levels in many crops. Consequently, the CARM model has the usual endogenous price structure based on linear crop demand functions which are estimated from time series data. The livestock sector is not included in the model.

The structure of the model is of a conventional quadratic form modified to accommodate a PQP implicit cost function for each region and crop. Average costs of production by region and resource input requirements are calculated from county level farm management data. Constraints on production are few since seasonal labor is generally available and agronomic crop rotation constraints are rare. Land and water availability are the dominant regional constraints on production. The objective function maximizes the sum of

producer and consumer surplus subject to the perfectly competitive marginal conditions holding for producers in each region. The PQP implicit cost represents the difference between the average and marginal value product per acre.

The CARM model is calibrated by the PQP method previously outlined. However we have been able to collect a time series of ten years of regional crop acreage and production parameters from 1973-1982. This substantial data set enables the model to be calibrated in a statistical manner which forms the basis of short run sectoral supply response projections. By regressing on nine years of cross-sectional data, factors affecting the systematic change in the dual can be estimated. The estimation of the PQP coefficient which exactly calibrates a model for a single year is analogous to a zero degree of freedom estimator, it always has a perfect fit, but its properties are suspect. Using a time series cross-section regression with the current PQP value as the dependent variable, substitutes a least squares criterion for the single period exact calibration. The resulting estimates are for more robust and yields a statistical basis for model projections.

The regressions were run as single equation weighted least squares. Each crop was regressed on the time series cross-sectional data from nine of the ten years available. The dependent variable is the crop dual value for a particular region and year. The explanatory variable specification is based on regional crop comparative advantage, partial adjustment of expected profits and indices of current annual profitability. The regional differences in crop yields and seasons are specified by dummy variable shifts in the equation intercepts. The one year lagged dual variable and two year lagged acreage captures the partial adjustment process of expectations. While the current price and cost indices reflect expectations on the changed crop returns in the

current year. In addition, a time trend and a dummy variable were included to reflect the drought condition that was known a priori in 1977.

Twenty-eight crop equations were estimated over 14 regions for eight years. There are 209 crop/region acreages observed in each year. The smallest number of regions growing a crop is found with celery, grown only in two areas, alfalfa, in contrast is grown in 13 of the 14 areas. The time series over which the regressions were fitted was a very turbulent one for California agriculture. 1974 to 1981 covered the period of a substantial change in the cost of all energy related inputs, a major drought in 1976 and 1977, substantial changes in crop export prices and government programs. The fluctuations in crop profitability are directly reflected in the PQP dual values, despite this volatility the 28 equations explained a large proportion of the variability. The specification and results for the 28 equations are detailed in Howitt 1985. Table 4 summarizes the fit of the equations.

TABLE 4

R² (CORRECTED) OF REGRESSIONS ON PQP DUALS

Range of R ²	Number	Percent
.999-.90	9	32
.899-.80	12	43
.799-.750	5	18
<.750	2	7
	<u>28</u>	

The explanatory values for the tenth year of the time series (1982) were used with the equations estimated from the previous years to forecast the dual values for 1982. The 28 equations yielded 209 forecasts for regional crop duals. The forecast PQP values were then used in the CARM model to predict regional acreage allocation by farmers in 1982.

The results for the statewide acreage predictions were under 30 percent absolute error for 19 of the 28 crops (Table 5). Of the nine crops whose errors exceeded 30 percent four were small acreage specialty crops.

TABLE 5
PREDICTED STATEWIDE CROP ACREAGE FOR 1982

Crop	Prediction Error Percent	Crop	Prediction Error Percent
Alfalfa	-2.5	Grain Sorghum	-7.1
Alfalfa Seed	-54.6	Lettuce	-1.1
Asparagus	4.7	Onions	109.8
Dryland Barley	-74.3	Irrigated Pasture	-2.7
Irrigated Barley	157.6	Potatoes	33.1
Beans	-30.1	Rice	-11.4
Broccoli	3.5	Safflower	33.1
Cantaloupes	2.2	Silage	8.6
Carrots	22.4	Strawberries	74.7
Cauliflower	5.1	Sugar Beet	-17.1
Celery	-2.6	Fresh Tomatoes	2.8
Corn	29.7	Processed Tomatoes	-20.0
Cotton	-51.4	Dryland Wheat	-47.7
Grain Hay	12.0	Irrigated Wheat	13.9

Over all crops the predicted statewide acreage underestimated the actual acreage by 4.4 percent.

As would be expected, the 209 regional predictions exhibited greater error than the statewide acreages. Table 6 summarizes the error magnitudes for the regional acreages.

TABLE 6
ACREAGE PREDICTION ERROR BY SUBREGION FOR 1982

Error Range Percent	Number of Regions	Percent
0-10	42	20
10-19.9	23	11
20-29.9	25	12
30-39.9	21	10
40-49.9	24	12
>50	74	35
	<u>209</u>	

The results (Table 5 and 6) show that for the current data base and prediction equations the model predictions can be considered validated by prediction at the statewide level, but not as yet at the local production level. We are optimistic that a longer time series and improved prediction equation specification will yield model validation at the production region level.

CONCLUSIONS

The results from both the TASM and CARM models show that agricultural sectoral and regional models can use the PQP method to successfully calibrate the model to single year or time series data.

Validation by predicting acreage allocation response outside the base year(s) used to calibrate the model was demonstrated by both the TASM and CARM model on a statewide basis.

The PQP/Econometric approach offers substantial potential for improved precision of prediction and rapid sequential updating as the availability of time series data improves.

FOOTNOTES

¹See Howitt and Mean [1985] and Goodman et al. [1985] for further discussions on this as well as its extension to quadratic programming.

²To alleviate the arbitrariness in "naive" flexibility constraint, more "sophisticated" flexibility constraint incorporating econometric techniques are also suggested. See for example Bawden [1968] and King [1968] in a discussion on Sharples and Schaller [1968] and Sahi and Craddock [1974].

³The terms calibration, verification and validation tend, in general, to be used interchangeably in the literature since they all eventually serve the purpose of modifying the model parameters or data to improve the base year solution.

⁴Risk costs are specified at the activity level, whereas the PQP coefficients are specified at the area level in TASM. The risk aversion coefficient is taken as one in the present version of the model.

⁵A detailed algebraic statement of the model can be found in Kasnakoglu and Howitt [1985]. Also see Le-Si, Scandizzo and Kasnakoglu [1983] for an earlier, linearized, non-PQP version of TASM.

⁶A detailed discussion of TASM data can be found in Kasnakoglu and Howitt [1985] and Le-Si, Scandizzo and Kasnakoglu [1983].

⁷In TASM, PQP terms are introduced for production technology and fallow activities to capture the implicit costs or benefits of using tractors vs animals and producing with fallow vs without fallow, which were not fully captured by the linear technology and costs.

⁸Some calibration for consistency was necessary in the first stage basically due to the nature of the data employed in TASM, which has been gathered from different sources for the interrelated area, production, and consumption series. The exact natures of the corrections are specified in Kasnakoglu and Howitt [1985].

⁹See Kasnakoglu and Howitt [1985] for further validation results and discussion.

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